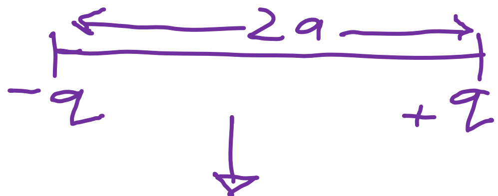
Electric Dipole

Electric Dipole: It is the pair of equal & opposite charges which is seperated by a small distance.



Electric dipole.

Dipole moment: It measures the strenght of electric dipole. It is the product of magnitude of either charge & distance b/w them. It is a vector quantity. It's Drn is from −ve to +ve.

$$\vec{p} = \text{Dipole moment} = q \times \vec{2a}$$

Drn: (−ve to +ve).

$$|\vec{p}| = |q| \times 2a = q \times 2a$$

Unit: cm or C−cm.

Ex: H_2O, HCl, C_2H_5OH, CH_3COOH etc. (all have permanent dipole moment).

Ideal or point Dipole: A dipole with neglible (small) size is called Ideal or point dipole.

Condition: $2a \to 0$ & $q \to \infty$. Such a way that Dipole moment has a finite value.

Characteristics: It is only specified by its Location & a dipole moment.

Ex: Invidual atoms or molecules.

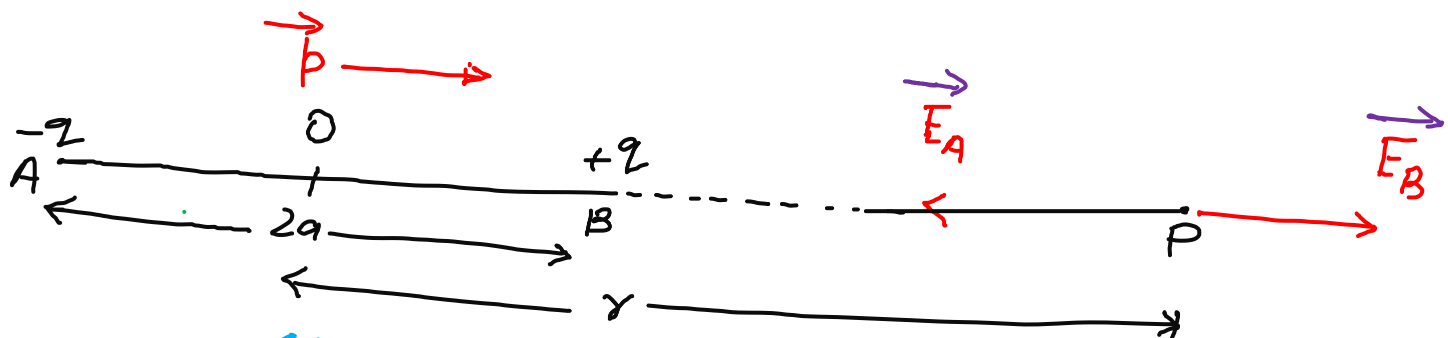
Dipole field: The field produced by electric dipole is called Dipole fied. The total Charge on electric dipole is zero. But the electric field of an electric dipole is not zero. It can be determined by using

(i) formula for the field of point charge.

(ii) The principle of Superposition.

Electric Field at an axial Point of a Dipole

Consider an Electric dipole Consisting of Charges $-q$ & q Seperated by a Small distance $2a$ & placed in Vacuum. Let 'p' be the Point on the axial line at distance r from the centre 'O' of the dipole on the side of charge $+q$.



electric field at an axial point of dipole.

E_A = Electric field at p due to charge $-q$.

E_B = Electric field at p due to charge $+q$.

then, $\vec{E}_A = \frac{1}{4\pi\epsilon_0} \frac{-q}{AP^2}$, along PA

$= \frac{1}{4\pi\epsilon_0} \frac{-q}{(r+a)^2}$, along PA.

& $\vec{E}_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{BP^2}$, along BP

$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-a)^2}$, along BP.

Hence Resultant of electric field at point P

$\vec{E}_{net} = \vec{E}_A + \vec{E}_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{-q}{(r+a)^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-a)^2}$

$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] = \frac{q}{4\pi\epsilon_0} \cdot \frac{4ar}{(r^2-a^2)}$, along PB

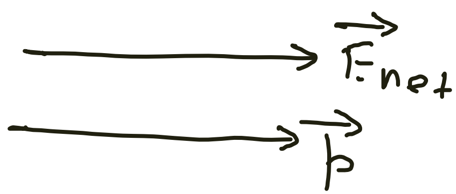
$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot 2r}{(r^2-a^2)^2}$, along PB $\quad [\because \vec{E}_B > \vec{E}_A]$

Or, $|\vec{E}_{net}| = E_{net} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2-a^2)^2}$, along PB if we assume $r >> a$, then a^2 can be neglected as compare to r^2.

then
$$E_{net} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{r^4} = \frac{2p}{4\pi\epsilon_0 r^3}$$

Note: Drⁿ of electric field due to axial lines will be same

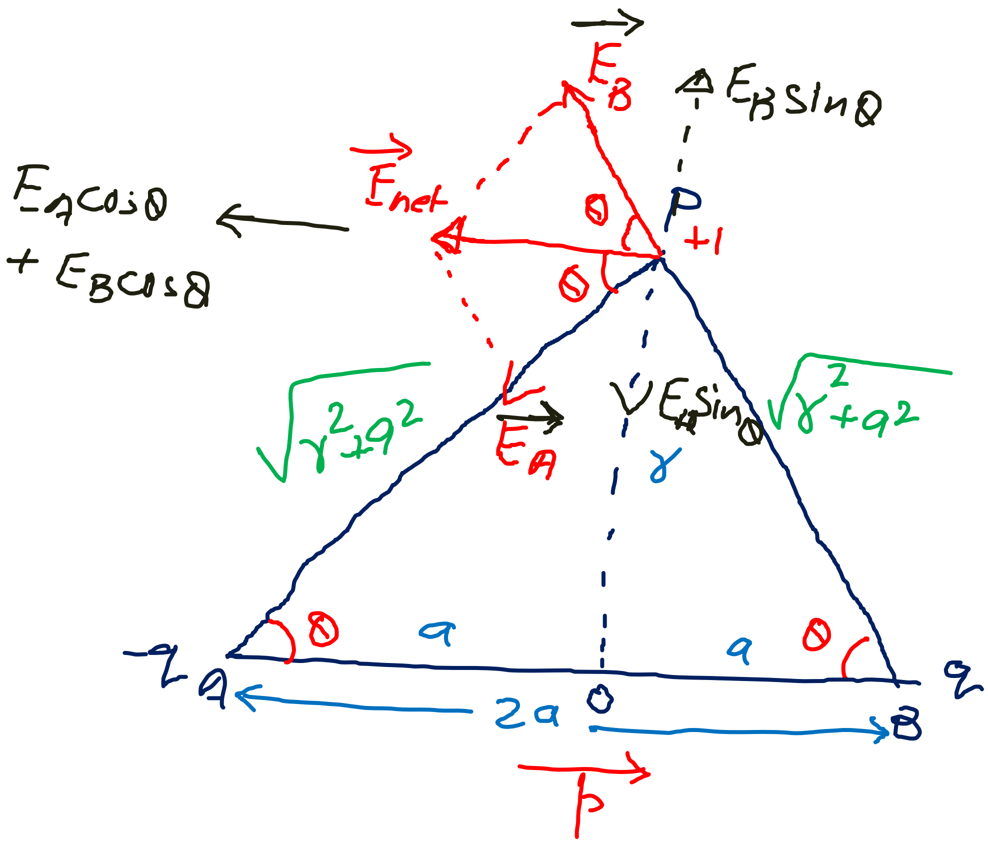
(i) the Drⁿ of Dipole moment. from -ve to posite.



(ii) Drⁿ of resultant always will be greater value of quantity $(E_B > E_A) \rightarrow$ take Drⁿ of (E_B) not E_A to resutant Drⁿ.

Electric field at an Equotorial point of a Dipole

Consider an Electric dipole consisting of charge -q & q which is seperated by a distance '2a' & placed in vacuum. Let point 'p' be a point on the Equotorial line of a dipole at a distance 'r' from it.



Where,

E_A = Electric field due to charge -q at A

E_B = Electric field due to charge -q at B.

then, Electric field at point P due to charge $-q$ at A

$$\vec{E_A} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)}$$, along PA

&

Electric field at point P due to charge $+q$ at B.

$$\vec{E_B} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)}$$, along BP

Magnitude of $|\vec{E_A}|$ & $|\vec{E_B}|$ are equal

$$\boxed{E_A = E_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)}}$$

Here, component of electric field E_A & E_B will be vertical & horizontal. Vertical will be canceled But Horizontal will be add up.

then, Total electric field at point 'p' will be sum of horizontal component in opposite to $\vec{p}$ Dr^n.

$$\vec{E}_{net} = \vec{E}_A \cos\theta + \vec{E}_B \cos\theta$$

or, $|\vec{E}_{net}| = E_A \cos\theta + E_B \cos\theta = 2E_A \cos\theta \quad [\because E_A = E_B]$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{1/2}} = \frac{2qa}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}$$

then, $E_{net} = \frac{q \times 2a}{4\pi\epsilon_0 (r^2+a^2)^{3/2}} = \frac{p}{4\pi\epsilon_0 (r^2+a^2)^{3/2}} \quad [\because p = q \times 2a]$

if the point p is Located far away from the dipole. then $r >> a$.

$E_{net} = \frac{p}{4\pi\epsilon_0 (r^2)^{3/2}} = \frac{p}{4\pi\epsilon_0 r^3}$, Dr^n will be Opposite to $\vec{p}$.

Note: (i) comparison B/W electric field short dipole at axial & equotorial will be

$$E_{axial} = \frac{2p}{4\pi\epsilon_0 r^3}$$

$$E_{equot.} = \frac{p}{4\pi\epsilon_0 r^3}$$

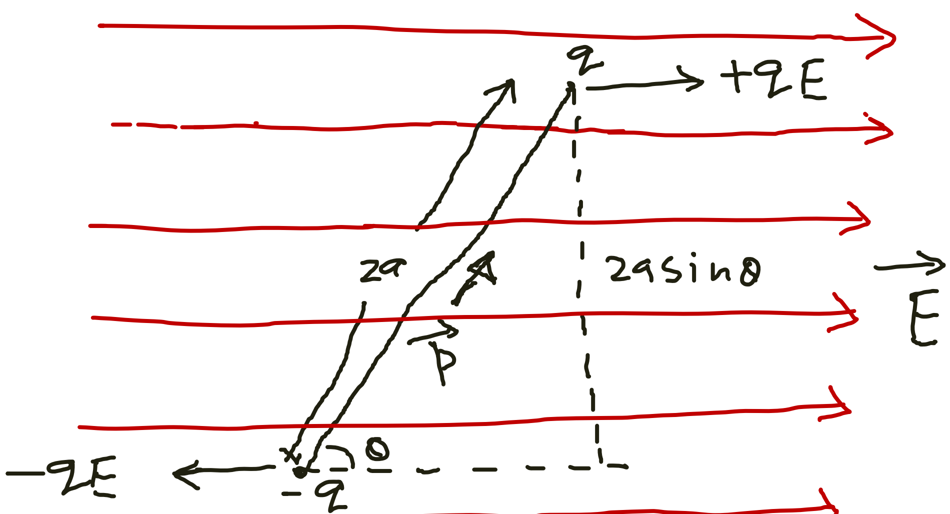
$$\Rightarrow \boxed{E_{axial} = 2E_{equotorial}}$$

(ii) Resultant Drn of electric field due to axial will be same as Drn of $\vec{p}$. But in case of equotrial it will be antiparallel to $\vec{p}$.

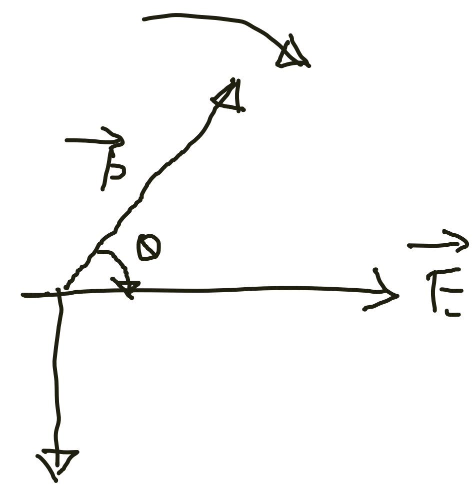
Torque on a dipole in a uniform Electric field

Consider an Electric dipole consisting of charge +q & -q & seperated distance 2a placeed in a uniform Electric field $\vec{E}$ making angle θ with it. It has dipole moment of magnitude

$$|\vec{p}| = q \times 2a$$



Torque on a Dipole in U E F.



$$\vec{\tau} = \vec{p} \times \vec{E}$$

Dr^n Right Hand screw rule.

Here, force exerted on charge +q by the field $\vec{E} = q\vec{E}$ (along $\vec{E}$)

force exerted on charge −q by the field $\vec{E} = -q\vec{E}$ (opposite to $\vec{E}$)

then, $\boxed{\vec{F}_{Total} = +q\vec{E} - q\vec{E} = 0}$

net force in case of dipole in uniform electric field is zero. But two equal & opposite forces act at different points of a dipole. They forms a couple with exerts a torque

Here, torque = either force × ⊥r distance b/w the two forces.

$$\tau = qE \times 2a\sin\theta = (q \times 2a)E\sin\theta = pE\sin\theta$$

$(p = q \times 2a)$

$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$ (as Dⁿ of $\vec{\tau}$ is ⊥r to Both $\vec{p}$ & $\vec{E}$).

Note: (i) Dipole will align same Drⁿ of $\vec{E}$ then $\theta = 0°$

then $\tau = pE\sin 0° = 0$.

(ii) Torque will be maxᵐ, when $\theta = 90°$

then $\tau = pE\sin 90° = pE$.

(iii) Dipole moment: $\tau = pE\sin\theta$

$\theta = 90°$, $E = 1$ unit, then $\boxed{\tau = p}$

The Torque acting on an electric dipole placed ⊥r to a uniform electric field of unit strenght.

Dipole in a non-uniform electric field

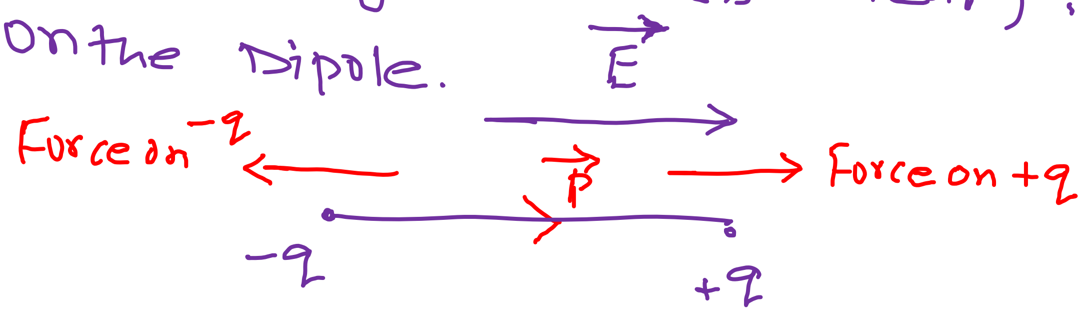
In non-uniform electric field dipole experiances different forces & not equal & Opposite. then, net force will not zero. & also torque depend on Location of dipole.

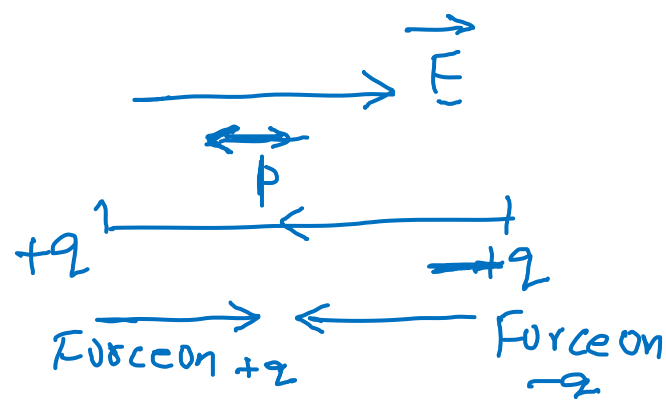
$$\vec{\tau} = \vec{p} \times \vec{E}(\vec{r})$$

where $\vec{r}$ is the position vector of the centre of the dipole.

When Dipole is parallel or antiparallel to $\vec{E}$.

Net torque on the Dipole is zero (because the force on the charges becomes linear). But there is net force on the Dipole.





Note: (i) uses in study of the effect of electric field on an insulator & in antenna analysis.

(ii) In uniform electric field an electric dipole experiances no net force but a non-zero torque.

(iii) In non-uniform electric field a dipole experiances a non-zero force & non-zero torque. But in case of parallel or antiparallel to the field the dipole experiances a zero torque & a non-zero force.

Summary

1. Dipole moment: $\vec{p} = |q| \times 2a$.

2. Dipole field due axial: $E_{axial} = \frac{2p}{4\pi\epsilon_0 r^3}$

3. Dipole field Equatorial: $E_{eq} = \frac{p}{4\pi\epsilon_0 r^3}$

$E_{axial} = 2E_{equa}$

4. Torque = $\vec{\tau} = \vec{p} \times \vec{E}$

$\tau = pE\sin\theta$.

5. Dipole field $\left(E \propto \frac{1}{r^3}\right)$ & electric field of a point charge $\left(E \propto \frac{1}{r^2}\right)$

Dipole field → decreases more rapidly

Problems Based on Dipole field & Torque

Q1. An electric dipole when held at 30° with respect to a electric field (uniform) 10^4 N/C Experiances a Torque 9×10^{-26} NM. Calculate dipole moment of a dipole.

Q2. Two charges $\pm 10 \mu C$ are placed 5.00 mm apart. Determine electric field at

(a) a point P on the axis of dipole 15 cm away from centre on the side of +ve charge.

(b) a point 'Q' 15 cm away from O on a line passing through O & normal to the axis of the dipole.